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HOW TO ANALYSE VECTOR-MESON PRODUCTION IN INELASTIC LEPTON SCATTERING

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Abstract: The kinematics of the process $\ell N \to \ell NV$, is studied in the one-photon approximation for unpolarized as well as polarized leptons ℓ. The vector-meson spin density matrix is expressed in terms of the s-channel helicity amplitudes in the hadron c.m.s. and the vector-meson decay angular distribution is discussed. The use of longitudinally polarized lepton beams is found to increase considerably the amount of information that can be deduced from the decay distribution. With longitudinal beam polarization it is possible to separate all 26 observable independent density matrix elements into contributions from natural and unnatural parity exchange in the t-channel, respectively.

1. Introduction

Leptoproduction experiments of vector-mesons, e.g.

$$\ell N \to \ell NV, \qquad V = \rho, \omega, \phi \tag{1}$$

are presently being performed at various laboratories [1–3]. They will provide interesting data on vector-meson decay and hence on the spin structure of reaction (1). Furthermore, with the advent of intense, highly energetic and polarized lepton (electron or muon) beams at SLAC, NAL and CERN II it will be possible to measure vector-meson production with polarized leptons. Motivated by these developments we attempt a systematic presentation of the vector-meson decay analysis in the one-photon exchange approximation, including lepton polarization. In view of muon scattering at small momentum transfers we avoid the usual zero-lepton mass approximation. We explore the maximum amount of information than can be obtained on the virtual process

$$\gamma N \to VN \tag{2}$$

from the hadronic decay of the vector-meson, with unpolarized and polarized leptons. We also discuss the separation of the observable vector-meson spin density matrix elements into contributions from natural and unnatural parity exchange in the t-channel. A complete separation is experimentally possible provided the measurements are carried out with longitudinally polarized leptons at different lepton scattering angles.

The present work is a continuation of a paper on vector-meson production by polarized photons [4]. We employ a unified notation and proper normalization so as to facilitate the comparison between real and virtual photoproduction.

Since we address ourselves primarily to the experimentalists the derivations are given in a fairly detailed manner. The paper aims at a comprehensive discussion of the subject and draws heavily on the works of Hand [5], Akerlof et al. [6] (photon density matrix $\rho(\gamma)$ for unpolarized leptons); Fraas and Schildknecht [8] (vector-meson decay for the case of s-channel helicity conservation); Dieterle [9] (general case of the vector-meson density matrix); Gourdin [10] (inclusive particle production by polarized leptons) and Chadwick [11] ($\rho(\gamma)$ without small lepton mass approximation).*

2. The photon density matrix

2.1. Basic notations

We consider vector-meson production in lepton-nucleon collisions, eq. (1), and denote, as indicated in fig. 1, by l_1, l_2, n_1, n_2, the four-momenta of the incoming and outgoing leptons and nucleons, respectively, and the vector-meson momentum by v. The four momentum of the virtual photon is defined as

$$q = l_1 - l_2 \,, \tag{3}$$

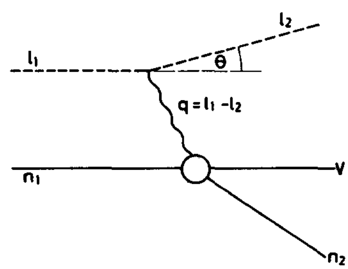

Fig. 1.

* After completion of this work we received a preprint by Fraas who treated the case of vector-meson production by polarized leptons on a polarized target.

$$Q^2 = -q^2 \; ; \tag{4}$$

Q^2 is positive for space-like photons.

Let us briefly list a few well-known formulae, which express the kinematics in terms of laboratory quantities:

$$Q^2 = Q^2_{\min} + 2\,|l_1|\,|l_2|\,(1 - \cos\Theta)\,,$$

$$Q^2_{\min} = -\,2m^2 + 2\,(l_{1\mathrm{o}} l_{2\mathrm{o}} - |l_1|\,|l_2|)\,, \tag{5}$$

where m is the lepton mass, Θ is the lepton scattering angle and $l_{i\mathrm{o}}$, l_i, $i = 1, 2$, refer to the time and space components of the four-vectors for the incident and scattered leptons. If $Q^2 \gg m^2$,

$$Q^2 \cong 4E_1 E_2 \sin^2 \tfrac{1}{2}\Theta\,, \tag{6}$$

(in standard notation, $E_i = l_{i\mathrm{o}}$). The energy of the virtual photon is commonly denoted by

$$\nu = E_1 - E_2\,, \tag{7}$$

and the effective mass of the final state hadron system by W:

$$s = W^2 = 2M\nu + M^2 - Q^2\,, \tag{8}$$

where M is the nucleon mass. The square of the four-momentum transfer between the incoming and outcoming nucleon is called t,

$$t = (n_1 - n_2)^2\,. \tag{9}$$

We introduce a coordinate system in the hadronic c.m.s. through the following orthogonal set of unit vectors:

$$Z = \frac{\boldsymbol{q}^*}{|\boldsymbol{q}^*|}\,, \quad Y = \frac{\boldsymbol{q}^* \times \boldsymbol{v}^*}{|\boldsymbol{q}^* \times \boldsymbol{v}^*|}\,, \quad X = Y \times Z\,. \tag{10}$$

The vector-meson production angle with respect to the direction of the virtual photon is denoted by θ^*_v,

$$\cos\theta^*_\mathrm{v} = \frac{\boldsymbol{q}^* \cdot \boldsymbol{v}^*}{|\boldsymbol{q}^*|\;|\boldsymbol{v}^*|}\,. \tag{11}$$

The angle Φ is defined as the angle between the normals to the lepton scattering plane

$$e_\ell = \frac{l_1^* \times l_2^*}{|l_1^* \times l_2^*|}, \tag{12}$$

and the hadron production plane, Y

$$\cos\Phi = e_\ell \cdot Y, \qquad \sin\Phi = \frac{((Y \times e_\ell) \times Y) \cdot e_\ell}{|(Y \times e_\ell) \times Y|}. \tag{13}$$

The decay distribution of the vector meson will be described in the vector meson rest frame using the helicity system with the z-axis opposite to the direction of the outgoing nucleon in the c.m.s.:

$$z = \frac{-n_2^*}{|n_2^*|}, \qquad y = Y, \qquad x = y \times z. \tag{14}$$

The decay angles θ, ϕ are defined as the polar and azimuthal angles of the unit vector π, which in case of a two-particle decay points along the momentum of one of the decay particles (or in case of a three-particle decay along the normal to the decay plane):

$$\cos\theta = \pi \cdot z, \qquad \cos\phi = \frac{y \cdot (z \times \pi)}{|z \times \pi|}, \qquad \sin\phi = -\frac{x \cdot (z \times \pi)}{|z \times \pi|} \tag{15}$$

2.2. The photon density matrix for unpolarized incident leptons

In the one-photon exchange approximation, the matrix element describing process (1) is of the form

$$M = e^2 \langle l_2 | j_\mu^{\mathrm{el}} | l_1 \rangle \cdot \langle n_2 v | j^{\mathrm{el}\mu} | n_1 \rangle = e^2 j_\mu \cdot J^\mu, \tag{16}$$

where $e^2/4\pi = \frac{1}{137}$; j_μ, J_μ are the matrix elements of the electromagnetic current operator j_μ^{el} sandwiched between the lepton and hadron states, respectively. The normalization is such that the differential cross section for vector meson production by eN scattering reads [6]:

$$\frac{d\sigma_{\mathrm{eN}\to\mathrm{eNV}}}{dE_2\, d\Omega_\ell\, d\Phi\, dt} = \frac{1}{(2\pi)^5} \frac{E_2}{E_1} \frac{m^2}{4(\nu^2 + Q^2)^{\frac{1}{2}}} \frac{1}{Q^4} \frac{1}{4} \sum_{\mathrm{spins}} |M|^2, \tag{17}$$

where $d\Omega_{\ell}$ is the volume element of the scattered lepton. With the abbreviations

$$\lambda_{\mu\nu} = m^2 \langle l_2 | j_\mu^{\mathrm{el}} | l_1 \rangle \langle l_2 | j_\nu^{\mathrm{el}} | l_1 \rangle^* , \tag{18}$$

$$t^{\mu\nu} = \langle n_2 v | j^{\mathrm{el}\mu} | n_1 \rangle \langle n_2 v | j^{\mathrm{el}\nu} | n_1 \rangle^* , \tag{19}$$

$|M|^2$ can be written in terms of the known leptonic tensor $\lambda_{\mu\nu}$ and a hadronic tensor $t_{\mu\nu}$

$$|M|^2 = \frac{e^4}{m^2} \lambda_{\mu\nu} t^{\mu\nu} . \tag{20}$$

The summation of $\lambda_{\mu\nu}$ over the lepton spins leads to

$$\widetilde{L}_{\mu\nu} = \sum_{\text{spins}} \lambda_{\mu\nu} = m^2 \operatorname{Tr} \left\{ \frac{(\not{l}_2 + m)}{2m} \gamma_\mu \frac{(\not{l}_1 + m)}{2m} \gamma_\nu \right\} \tag{21}$$

$$= (l_{1\mu} l_{2\nu} + l_{2\mu} l_{1\nu} - \tfrac{1}{2} Q^2 g_{\mu\nu}) .$$

Defining correspondingly a tensor $\widetilde{T}_{\mu\nu}$

$$\widetilde{T}^{\mu\nu} = \sum_{\text{spins}} t^{\mu\nu} , \tag{22}$$

the final form reads

$$\tfrac{1}{4} \sum_{\text{spins}} |M|^2 = \tfrac{1}{4} \widetilde{L}_{\mu\nu} \widetilde{T}^{\mu\nu} . \tag{23}$$

Without loss of generality we evaluate $\widetilde{L}_{\mu\nu}$ in a coordinate system, with the z-axis pointing along $\boldsymbol{q} = l_1 - l_2$ and l_1, l_2 in the x, z plane, i.e. $l_{1y} = l_{2y} = 0$.

The tensor $\widetilde{L}_{\mu\nu}$ describes the photon spin state and therefore can be called the spin density matrix of the photon. It has in general transverse (x, y) as well as longitudinal (z) and scalar (0) components. While the transverse polarizations remain unchanged under Lorentz transformation along $\boldsymbol{q}$, the longitudinal and scalar components transform into each other. It is therefore customary to treat them as a single entity. We eliminate the scalar component through current conservation

$$q_\mu j^\mu = 0 , \qquad q_\mu J^\mu = 0 , \tag{24}$$

and obtain as a result

$$j_\mu J^\mu = j_x J_x + j_y J_y - \frac{Q^2}{q_0^2} j_z J_z \ . \tag{25}$$

This is equivalent to the formal replacement

$$j_\mu = (j_x, j_y, j_z, j_0) \to j'_\mu = (j_x, j_y\ , -\frac{Q}{q_0} j_z, 0)\ , \tag{26}$$

$$J_\mu = (J_x, J_y, J_z, J_0) \to J'_\mu = (J_x, J_y\ , +\frac{Q}{q_0} J_z, 0)\ ,$$

where the powers of Q/q_0 $(Q \equiv \sqrt{Q^2})$ have been assigned to j_z and J_z in such a way, that

$$\sum_{\mu=3,4} j_\mu j^\mu = - \sum_{\mu=3,4} j'_\mu j'^\mu\ , \tag{27}$$

and similarly for J_μ, J'_μ. Thus the replacement does not alter the longitudinal-scalar flux.

Insertion of j', J' into eqs. (18), (19), (21), (22) leads to a replacement of $\widetilde{L}_{\mu\nu}$, $\widetilde{T}_{\mu\nu}$ by the 3 X 3 matrices $L_{\mu\nu}$, $T_{\mu\nu}$:

$$L_{\mu\nu} = \begin{cases} \widetilde{L}_{\mu\nu} & \mu, \nu = 1, 2\ ; \\ -\left(\frac{Q}{q_0}\right)\widetilde{L}_{\mu 3} & \mu = 1, 2, \quad \nu = 3\ ; \\ -\left(\frac{Q}{q_0}\right)\widetilde{L}_{3\nu} & \mu = 3\ , \quad \nu = 1, 2\ ; \\ \left(\frac{Q}{q_0}\right)^2 \widetilde{L}_{33} & \mu = \nu = 3\ . \end{cases} \tag{28}$$

$$T_{\mu\nu} = \begin{cases} \widetilde{T}_{\mu\nu} & \mu, \nu = 1, 2\ ; \\ \left(\frac{Q}{q_0}\right)\widetilde{T}_{\mu 3} & \mu = 1, 2\ , \quad \nu = 3\ ; \\ \left(\frac{Q}{q_0}\right) T_{3\nu} & \mu = 3, \quad \nu = 1, 2\ ; \\ \left(\frac{Q}{q_0}\right)^2 T_{33} & \mu = \nu = 3\ . \end{cases} \tag{29}$$

Eq. (23) becomes now:

$$\frac{1}{4}\sum_{\text{spins}} |M|^2 = \frac{1}{4}\sum_{\text{spins}} L_{\mu\nu} T^{\mu\nu} . \tag{30}$$

The polarization of a physical spin 1 particle is invariant under a Lorentz boost. Inspection of $L_{\mu\nu}$ shows that this is true also for the photon spin density matrix. For the transverse components the invariance property is obvious; we give the proof for L_{13}. Current conservation yields

$$L_{13} = -\frac{Q}{q_0}\widetilde{L}_{13} = -\frac{Q}{q_3}\widetilde{L}_{10} . \tag{31}$$

From eq (21) and observing that in the coordinate system chosen $l_{11} = l_{21}$, we have

$$\begin{aligned} \widetilde{L}_{13} &= l_{11}(l_{13} + l_{23}) , \\ \widetilde{L}_{10} &= l_{11}(l_{10} + l_{20}) . \end{aligned} \tag{32}$$

We apply a Lorentz transformation $L(\beta)$ along $\boldsymbol{q}$ on $\widetilde{L}$. The result is, with $\gamma = (1-\beta^2)^{-\frac{1}{2}}$

$$\begin{aligned} L'_{13} &= -\frac{Q}{q'_3}\widetilde{L}'_{10} = \frac{-Q\cdot l_{11}}{\gamma(q_3+\beta q_0)}\gamma\{(l_{10}+l_{20})+\beta(l_{13}+l_{23})\} \\ &= -Ql_{11}\frac{(l_{10}+l_{20})}{q_3}\frac{1}{1+\beta\dfrac{q_0}{q_3}}\left\{1+\beta\frac{(l_{13}+l_{23})}{(l_{10}+l_{20})}\right\} \\ &= \frac{-Q}{q_3}l_{11}(l_{10}+l_{20}) = L_{13} . \end{aligned} \tag{33}$$

Since $L_{\mu\nu}$ is invariant under boosts along $\boldsymbol{q}$ we may choose a suitable system to evaluate $L_{\mu\nu}$. This is the Breit system, characterized by $l_{10} = l_{20}$ (see fig. 2) i.e. $l_1 = (l_x, 0, l_z, l_0)$, $l_2 = (l_x, 0, -l_z, l_0)$, $q = (0, 0, 2l_z = q, 0)$.

Eq. (28) has to be rewritten before being applicable in the Breit system, since here $q_0 = 0$. For example,

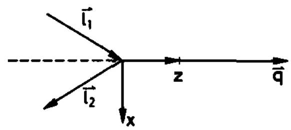

Fig. 2.

$$L_{13} = -\frac{Q}{q_0}(l_{1x}l_{2x} + l_{2x}l_{1z}) = -\frac{Q}{q_0} l_x (l_{1z} + l_{2z})$$

$$= -\frac{Q}{q_0} l_x \frac{l_{1z}^2 - l_{2z}^2}{l_{1z} - l_{2z}} = -\frac{Ql_x}{q_0} \frac{l_{10}^2 - l_{20}^2}{l_{1z} - l_{2z}} = -Ql_x \frac{l_{10} - l_{20}}{q_0} \frac{l_{10} + l_{20}}{q_z} \qquad (34)$$

$$= -Q\frac{l_x}{q_z}(l_{10} + l_{20}) .$$

Evaluation in the Breit system gives

$$L_{13} = -2l_x l_0 . \qquad (35)$$

The remaining elements of $L_{\mu\nu}$ are found in the same way.

$$L_{\mu\nu} = 2 \begin{pmatrix} l_x^2 + \frac{1}{4}Q^2 & 0 & -l_x\sqrt{l_x^2 + m^2 + \frac{1}{4}Q^2} \\ 0 & \frac{1}{4}Q^2 & 0 \\ -l_x\sqrt{l_x^2 + m^2 + \frac{1}{4}Q^2} & 0 & l_x^2 + m^2 \end{pmatrix} \qquad (36)$$

It is customary to express $L_{\mu\nu}$ in terms of the polarization parameter ϵ, defined by

$$\frac{L_{11}}{L_{12}} = \frac{1+\epsilon}{1-\epsilon}, \qquad (37)$$

which gives

$$\epsilon = \left(1 + \frac{Q^2}{2l_x^2}\right)^{-1} . \qquad (38)$$

Furthermore, following Chadwick [11] for the case $m \neq 0$ a mass correction parameter δ is introduced,

$$\delta = \frac{2m^2}{Q^2}(1-\epsilon)\,. \tag{39}$$

Inserting eqs. (38), (39) we have

$$L_{\mu\nu} = \frac{Q^2}{1-\epsilon}\begin{pmatrix} \frac{1}{2}(1+\epsilon) & 0 & -\sqrt{\frac{1}{2}\epsilon(1+\epsilon+2\delta)} \\ 0 & \frac{1}{2}(1-\epsilon) & 0 \\ -\sqrt{\frac{1}{2}\epsilon(1+\epsilon+2\delta)} & 0 & \epsilon+\delta \end{pmatrix}. \tag{40}$$

This expression for $L_{\mu\nu}$ is also valid in the hadron c.m.s. system. Finally $L_{\mu\nu}$ will be transformed into the hadron c.m.s. helicity frame. This is accomplished by a transition from cartesian to spherical coordinates followed by a rotation around Z through the angle Φ (see eqs. (10), (13)):

$$L_{\lambda\lambda'} = U_{\lambda\mu} L_{\mu\nu} U^{-1}_{\nu\lambda'}\,, \tag{41}$$

where λ, λ' denote the photon helicity ($\lambda, \lambda' = 1, 0, -1$) and

$$U_{\lambda\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{e}^{-i\Phi} & -i\mathrm{e}^{-i\Phi} & 0 \\ 0 & 0 & \sqrt{2} \\ \mathrm{e}^{i\Phi} & -i\mathrm{e}^{i\Phi} & 0 \end{pmatrix}, \tag{42}$$

$$U^{-1}_{\mu\lambda} = U^{*}_{\lambda\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{e}^{i\Phi} & 0 & \mathrm{e}^{-i\Phi} \\ i\mathrm{e}^{i\Phi} & 0 & i\mathrm{e}^{-i\Phi} \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \tag{43}$$

The resulting photon spin density matrix reads

$$L_{\lambda\lambda'} = \frac{Q^2}{2(1-\epsilon)}\begin{pmatrix} 1 & \sqrt{\epsilon(1+\epsilon+2\delta)}\,\mathrm{e}^{-i\Phi} & -\epsilon\mathrm{e}^{-2i\Phi} \\ \sqrt{\epsilon(1+\epsilon+2\delta)}\,\mathrm{e}^{i\Phi} & 2(\epsilon+\delta) & -\sqrt{\epsilon(1+\epsilon+2\delta)}\,\mathrm{e}^{-i\Phi} \\ -\epsilon\mathrm{e}^{2i\Phi} & -\sqrt{\epsilon(1+\epsilon+2\delta)}\,\mathrm{e}^{i\Phi} & 1 \end{pmatrix} \tag{44}$$

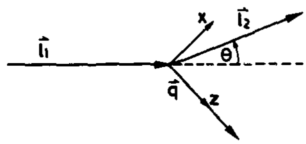

Fig. 3.

Let us now express ϵ in terms of lab system variables: we start by calculating the x component l_x of the lepton momentum (see fig. 3),

$$l_x = l_{1x} = \frac{l_1 \cdot (q \times l_2 \times q)}{|q \times l_2 \times q|} = \frac{l_1 \cdot \{l_2 q^2 - (q \cdot l_2) q\}}{\sqrt{l_2^2 q^4 + (q l_2)^2 q^2 - 2(q l_2)^2 q^2}}$$

$$= \frac{1}{|q|} \sqrt{l_1^2 l_2^2 - (l_1 \cdot l_2)^2} = \frac{1}{|q|} |l_1| \, |l_2| \sin \Theta \,. \tag{45}$$

Combining this result with eq. (5),

$$Q^2 - Q^2_{\min} = 2 |l_1| \, |l_2| \sin \Theta \tan \tfrac{1}{2}\Theta \,, \tag{46}$$

we find from eq. (38),

$$\epsilon = \left\{ 1 + 2 \frac{Q^2 + \nu^2}{Q^2 \left(1 - \dfrac{Q^2_{\min}}{Q^2}\right)^2} \tan^2 \tfrac{1}{2}\Theta \right\}^{-1} \tag{47}$$

which for $Q^2 \gg m^2$ reduces to the well known expression

$$\epsilon \simeq \left\{ 1 + 2 \frac{(Q^2 + \nu^2)}{Q^2} \tan^2 \tfrac{1}{2}\Theta \right\}^{-1} \tag{48}$$

2.3. The photon density matrix for polarized incident leptons

In order to describe a polarized lepton beam we shall start from a covariant projection operator [12] operating on Dirac spinors and employ the same trace technique applied to evaluate $L_{\mu\nu}$. We introduce the lepton polarization vector P_μ which in the lepton rest frame can be written as

$$P^0_\mu = P(\cos\alpha_1 \sin\alpha_2, \sin\alpha_1 \sin\alpha_2, \cos\alpha_2, 0)\,, \tag{49}$$

where the lepton is assumed to move in the z-direction. The angles α_1, α_2 characterize the orientation of the polarization vector: $\alpha_2 = 0, \pi$ corresponds to longitudinally, $\alpha_2 = \pi/2$ to transversely polarized leptons. The quantity P measures the degree of polarization, $0 \leqslant P \leqslant 1$. The covariant density matrix of a polarized lepton beam can be written as [12]

$$\rho(\ell) = \tfrac{1}{2}(1 + \gamma_5 P_\mu \gamma^\mu) = \tfrac{1}{2}(1 + \gamma_5 \not{P})\,, \tag{50}$$

where P_μ is obtained from P^0_μ by a proper Lorentz transformation. For $P = 1$, $\rho(\ell)$ is a projection operator.

The cross section is calculated by taking the proper initial polarization state and averaging over the final state polarizations. Instead of $\widetilde{L}_{\mu\nu}$ we have a new matrix $\widetilde{\mathcal{L}}_{\mu\nu}$ [12]

$$\begin{aligned}\widetilde{\mathcal{L}}_{\mu\nu} &= \tfrac{1}{2}\,\mathrm{Tr}\,\{(\not{l}_2 + m)\,\gamma_\mu(\not{l}_1 + m)\tfrac{1}{2}(1 + \gamma_5 \not{P})\,\gamma_\nu\} \\ &= \widetilde{L}_{\mu\nu} + \frac{m}{4}\,\mathrm{Tr}\,\{\gamma_\mu \not{l}_1 \gamma_5 \not{P} \gamma_\nu + \not{l}_2 \gamma_\mu \gamma_5 \not{P} \gamma_\nu\}\,.\end{aligned} \tag{51}$$

The polarization dependent part is

$$\begin{aligned}\widetilde{M}_{\mu\nu} \equiv \widetilde{\mathcal{L}}_{\mu\nu} - \widetilde{L}_{\mu\nu} &= -\frac{m}{4}\,\mathrm{Tr}\,\gamma_5\;\{(\not{l}_1 - \not{l}_2)\,\gamma_\mu \not{P} \gamma_\nu - 2 l_{1\mu} \not{P} \gamma_\nu\} \\ &= im\,\epsilon_{\mu\nu\rho\sigma}\,q^\rho P^\sigma\,,\end{aligned} \tag{52}$$

where the tensor $\epsilon_{\rho\mu\sigma\nu}$ is defined as

$$\epsilon_{\rho\mu\sigma\nu} = \begin{cases} \pm 1 & \text{if } (\rho, \mu, \sigma, \nu) \text{ is an } \binom{\text{even}}{\text{odd}} \text{ permutation of } (1, 2, 3, 4)\,, \\ 0 & \text{if two indices are equal}\,. \end{cases}$$

As in the case of $\widetilde{L}_{\mu\nu}$ we replace $\widetilde{M}_{\mu\nu}$ by a 3×3 matrix $M_{\mu\nu}$,

$$M_{\mu\nu} = \begin{cases} \widetilde{M}_{\mu\nu} & \mu, \nu = 1, 2\,, \\ -\dfrac{Q}{q_0}\,\widetilde{M}_{\mu 3} & \mu = 1, 2, \nu = 3\,, \\ -\dfrac{Q}{q_0}\,\widetilde{M}_{3\nu} & \mu = 3, \quad \nu = 1, 2\,, \end{cases} \tag{53}$$

and obtain

$$M_{\mu\nu} = im \begin{pmatrix} 0 & q_3 P_0 - q_0 P_3 & -QP_2 \\ -(q_3 P_0 - q_0 P_3) & 0 & QP_1 \\ QP_2 & -QP_1 & 0 \end{pmatrix} \tag{54}$$

The matrix $M_{\mu\nu}$ is invariant under boosts along $\boldsymbol{q}$. This is self-evident for all elements but M_{12} ($= -M_{21}$) for which the proof runs as follows:

$$\begin{aligned} M'_{12} &= \gamma(q_3 + \beta q_0)\gamma\,(P_0 + \beta P_3) - \gamma(q_0 + \beta q_3)\gamma(P_3 + \beta P_0) \\ &= \gamma^2 (1 - \beta^2)\,(q_3 P_0 - q_0 P_3) = q_3 P_0 - q_0 P_3 = M_{12}\,. \end{aligned} \tag{55}$$

Evaluation of $M_{\mu\nu}$ in the Breit system gives:

$$M_{\mu\nu} = imQ \begin{pmatrix} 0 & P_0 & -P_2 \\ -P_0 & 0 & P_1 \\ P_2 & -P_1 & 0 \end{pmatrix} \tag{56}$$

The transformation into the helicity basis yields (see eq. (41)):

$$M_{\lambda\lambda'} = U_{\lambda\mu} M_{\mu\nu}\, U^{-1}_{\mu\lambda'}\,,$$

$$M_{\lambda\lambda'} = mQ \begin{pmatrix} P_0 & \frac{1}{\sqrt{2}}(P_1 + iP_2)\mathrm{e}^{-i\Phi} & 0 \\ \frac{1}{\sqrt{2}}(P_1 - iP_2)\mathrm{e}^{i\Phi} & 0 & \frac{1}{\sqrt{2}}(P_1 + iP_2)\mathrm{e}^{-i\Phi} \\ 0 & \frac{1}{\sqrt{2}}(P_1 - iP_2)\mathrm{e}^{i\Phi} & -P_0 \end{pmatrix} \tag{57}$$

In the above relations the lepton polarization vector P_μ is to be taken in the Breit system. In a last step we express P_μ in terms of its rest system parameters (eq. (49)) where the z-axis was chosen along the direction of the lepton momentum in the lab. system. This requires firstly a Lorentz transformation from the lepton rest system into the Breit system. The parameters γ, β of the transformation can be read off from the lepton four momentum vector in the Breit system:

$$\gamma = \frac{\sqrt{l_x^2 + \frac{1}{4}Q^2 + m^2}}{m} = \frac{Q}{2m} \sqrt{\frac{1+\epsilon+2\delta}{1-\epsilon}}\,, \tag{58}$$

$$\gamma\beta = \frac{\sqrt{l_x^2 + \frac{1}{4}Q^2}}{m} = \frac{Q}{2m} \sqrt{\frac{1+\epsilon}{1-\epsilon}}\,.$$

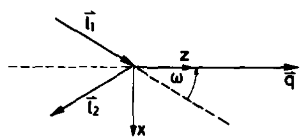

Fig. 4.

Secondly we rotate P_μ around the y-axis so as to have $\boldsymbol{q}$ pointing along the z-direction (see fig. 4). The rotation angle ω is given by

$$\sin\omega = \frac{l_x}{|l_1|} = \sqrt{\frac{2\epsilon}{1+\epsilon}}\,. \tag{59}$$

This leads to the final result:

$$\begin{aligned} P_\mu^{\mathrm{BS}} = P \Bigg(& \sqrt{\frac{1-\epsilon}{1+\epsilon}} \cos\alpha_1 \sin\alpha_2 + \frac{Q}{2m} \sqrt{\frac{2\epsilon(1+\epsilon+2\delta)}{1-\epsilon^2}} \cos\alpha_2\,, \\ & \sin\alpha_1 \sin\alpha_2\,, \\ & \sqrt{\frac{2\epsilon}{1+\epsilon}} \cos\alpha_1 \sin\alpha_2 + \frac{Q}{2m} \sqrt{\frac{1+\epsilon+2\delta}{1+\epsilon}} \cos\alpha_2\,, \\ & \frac{Q}{2m} \sqrt{\frac{1+\epsilon}{1-\epsilon}} \cos\alpha_2 \Bigg)\,, \end{aligned} \tag{60}$$

which for $Q^2 \gg m^2$ can be approximated by

$$P_\mu^{\mathrm{BS}} \simeq \frac{Q}{2m} P \left(\sqrt{\frac{2\epsilon}{1-\epsilon}} \cos\alpha_2\,, 0\,, \cos\alpha_2\,, \sqrt{\frac{1+\epsilon}{1-\epsilon}} \cos\alpha_2 \right). \tag{61}$$

We see that for purely longitudinally polarized leptons ($\alpha_2 = 0, \pi$), $P_2^{\mathrm{BS}} = 0$. The other components will be large provided $Q > m$. For purely transversally polarized leptons there will be no large polarization effects for $Q \gg m$, since the enhancing factor Q/m is absent.

Eq. (60) can be inserted into eq. (57) to evaluate $M_{\lambda\lambda'}$ in the Breit system. In the limit $Q^2 \gg m^2$ the result is

$$M_{\lambda\lambda'} = \frac{Q^2}{2\sqrt{1-\epsilon}} P \cos\alpha_2 \begin{pmatrix} \sqrt{1+\epsilon} & \sqrt{\epsilon}\, \mathrm{e}^{-i\Phi} & 0 \\ \sqrt{\epsilon}\, \mathrm{e}^{i\Phi} & 0 & \sqrt{\epsilon}\, \mathrm{e}^{-i\Phi} \\ 0 & \sqrt{\epsilon}\, \mathrm{e}^{i\Phi} & -\sqrt{1+\epsilon} \end{pmatrix}$$

2.4. Decomposition of the photon density matrix

The photon density matrix, $\rho(\gamma)$, normalized to unit flux of transverse photons, is given by

$$\rho(\gamma)_{\lambda\lambda'} = \frac{1-\epsilon}{Q^2} \{ L_{\lambda\lambda'} + M_{\lambda\lambda'} \} . \tag{62}$$

Note that the flux of longitudinal photons is in the ratio $\Gamma_L/\Gamma_T = \epsilon + \delta$ relative to the transverse flux (see eq. (44)). In analogy to the case of photoproduction [4] we decompose $\rho(\gamma)$ into an orthogonal set of independent hermitian matrices Σ^α

$$\rho(\gamma) = \tfrac{1}{2} \sum_{\alpha=0}^{8} \tilde{\Pi}_\alpha \Sigma^\alpha ; \tag{63}$$

$$\Sigma^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \Sigma^1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \Sigma^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix};$$

$$\Sigma^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \quad \Sigma^4 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Sigma^5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\Sigma^6 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}; \Sigma^7 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \Sigma^8 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{64}$$

The vector $\widetilde{\Pi}$ has the following components:

$$\widetilde{\Pi} = \{1, -\epsilon \cos 2\Phi, -\epsilon \sin 2\Phi, \frac{2m}{Q}(1-\epsilon)P_0, \epsilon + \delta,$$
$$\sqrt{2\epsilon(1+\epsilon+2\delta)} \cos\Phi, \sqrt{2\epsilon(1+\epsilon+2\delta)} \sin\Phi,$$
$$\frac{2m}{Q}(1-\epsilon)(P_1 \cos\Phi + P_2 \sin\Phi), \frac{2m}{Q}(1-\epsilon)(P_1 \sin\Phi - P_2 \cos\Phi)\}\,. \tag{65}$$

The decomposition eq. (63) is unique because of the orthogonality relation,

$$\frac{\mathrm{Tr}\,(\Sigma^\alpha \Sigma^\beta)}{\mathrm{Tr}\,(\Sigma^\alpha \Sigma^\alpha)} = \delta_{\alpha\beta}\,,$$

and is equivalent to a separation of $\rho(\gamma)$ into vector and tensor polarization parts [13].

Table 1
Symmetry properties of the matrices Σ^α

Σ^α	$\Sigma^\alpha_{\lambda\lambda'} = C_\alpha(-1)^{\lambda-\lambda'}\Sigma^\alpha_{-\lambda-\lambda'}$	$\Sigma^\alpha_{\lambda\lambda'} = d_\alpha(-1)^{\lambda'}\Sigma^\beta_{\lambda-\lambda'}$	
	C_α	β	d_α
Σ^0	1	1	-1
Σ^1	1	0	-1
Σ^2	-1	3	i
Σ^3	-1	2	$-i$
Σ^4	1	4	1
Σ^5	1	5	1
Σ^6	-1	7	i
Σ^7	-1	6	$-i$
Σ^8	1	8	1

The matrices $\Sigma^0 - \Sigma^3$ describe transverse photons and correspond to those used in photoproduction [4]; Σ^0 gives the unpolarized part, Σ^1 and Σ^2 represent linear polarization, and Σ^3 circular polarization. The matrix Σ^4 describes longitudinal photons; $\Sigma^5 - \Sigma^8$ represent transverse/longitudinal interference terms.

We now turn to the symmetry properties of the matrices L, M and Σ^α which determine the structure of the vector meson decay distributions and also the separation into contributions from natural and unnatural parity exchange.

The matrices $L_{\lambda\lambda'}$ and $M_{\lambda\lambda'}$ have opposite symmetry under reflection at the antidiagonal:

$$L_{-\lambda-\lambda'} = (-1)^{\lambda-\lambda'} L_{\lambda'\lambda}, \tag{66}$$

$$M_{-\lambda-\lambda'} = -(-1)^{\lambda-\lambda'} M_{\lambda'\lambda}. \tag{67}$$

Correspondingly the matrices Σ^3, Σ^7 and Σ^8 obey relation (67) while the rest of the Σ^α follow relation (66). Further symmetry properties of the Σ^α are listed in Table 1.

3. Vector-meson production

3.1. The vector-meson density matrix

The previous section dealt with the lepton vertex and provided us with the density matrix of the photon. We now turn to the hadron vertex which represents the virtual process $\gamma_V \mathrm{N} \to \mathrm{VN}$. It is customary to express the cross section for this process in terms of $\mathcal{M}$ in the following manner:

$$\frac{d\sigma_{\gamma_V \mathrm{N}\to \mathrm{VN}}}{dt\, d\Phi} = \frac{1}{32\pi^2 (\nu^2 + Q^2)} \frac{2(1-\epsilon)}{Q^2} \tfrac{1}{4} \sum |\mathcal{M}|^2 . \tag{68}$$

Apart from the factor $2(1-\epsilon)/Q^2$ this corresponds to the usual expression for the photoproduction cross section (replacing j_μ by ϵ_μ in $\mathcal{M}$). The extra factor has been added to normalize to unit transverse photon flux (see eq. 44)).

Rather than using $\mathcal{M}$ we shall work with the standard formalism for two-body reactions by introducing the helicity amplitudes of Jacob and Wick [14] for $\gamma_V \mathrm{N} \to \mathrm{VN}$: $T_{\lambda_V \lambda_{N'} \lambda_\gamma \lambda_N}$ where the λ's denote the helicities of the particles. The connection between T and $\mathcal{M}$ is established through the relation

$$T_{\lambda_V \lambda_{N'} \lambda_\gamma \lambda_N} = \langle \lambda_V \lambda_{N'} | j_{\lambda_\gamma} | \lambda_N \rangle , \tag{69}$$

where

$$\langle \lambda_V \lambda_{N'} | j_{\pm 1} | \lambda_N \rangle = \frac{\mp 1}{\sqrt{2}} \langle \lambda_V \lambda_{N'} | j_x \pm i j_y | \lambda_N \rangle ,$$

and

$$\langle \lambda_V \lambda_{N'} | j_{\lambda\gamma=0} | \lambda_N \rangle = \langle \lambda_V \lambda_{N'} | j_z | \lambda_N \rangle .$$

In terms of T the cross section reads

$$\frac{d\sigma_{\gamma_V N \to VN}}{dt\, d\Phi} = \frac{1}{32\pi^2 (\nu^2 + Q^2)} \; \tfrac{1}{2} \operatorname{Tr} (T \rho(\gamma) T^+) . \tag{70}$$

Eq. (70) explicitly shows the dependence of $d\sigma_{\gamma_V N \to VN}/dt\, d\Phi$ on the photon polarization. In particular, after integration over Φ, eq. (70) yields

$$\frac{d\sigma_{\gamma_V N \to VN}}{dt} = \frac{d\sigma_T(\gamma_V N \to VN)}{dt} + (\epsilon + \delta) \frac{d\sigma_L(\gamma_V N \to VN)}{dt} ,$$

where $d\sigma_T/dt$ and $d\sigma_L/dt$ denote the cross sections for vector meson production by transverse and longitudinal photons respectively.

The helicity amplitudes as a consequence of parity conservation, obey the following symmetry relation (in the coordinate system chosen):

$$T(\Theta_V^*)_{-\lambda_V - \lambda_{N'}, -\lambda_\gamma - \lambda_N} = (-1)^{(\lambda_V - \lambda_{N'}) - (\lambda_\gamma - \lambda_N)} \, T(\Theta_V^*)_{\lambda_V \lambda_{N'}, \lambda_\gamma \lambda_N} , \tag{72}$$

with Θ_V^* being the production angle in the hadron c.m.s.

The vector-meson density matrix will be derived starting from the von-Neumann formula,

$$\tilde{\rho}(V) = \tfrac{1}{2} T \rho(\gamma) T^+ , \tag{73}$$

where summation over nucleon helicities is understood. For practical purposes it is convenient to work with the normalized matrix.

$$\rho(V) = \tilde{\rho}(V) / \int \frac{d\Phi}{2\pi} \operatorname{Tr} \tilde{\rho}(V) \tag{74}$$

where an averaging over Φ is being done.

The remaining part of this section deals with the decomposition of $\rho(\mathrm{V})$ into hermitian matrices suggested by the corresponding decomposition of $\rho(\gamma)$:

$$\rho(\mathrm{V}) = \sum_{\alpha=0}^{8} \Pi_\alpha \rho^\alpha , \tag{75}$$

with

$$\rho^\alpha_{\gamma_\mathrm{V}\gamma'_\mathrm{V}} = \frac{1}{2N_\alpha} \sum_{\lambda_{\mathrm{N}'},\lambda_\mathrm{N},\lambda_\gamma,\lambda'_\gamma} T_{\lambda_\mathrm{V}\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_\mathrm{N}} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'_\mathrm{V}\lambda_{\mathrm{N}'},\lambda'_\gamma\lambda_\mathrm{N}} . \tag{76}$$

Here the N_α are normalization constants which for the purely transverse parts ($\alpha = 0-3$) we choose to be identical to those used in photoproduction [4]. This will facilitate the comparison between photo- and electroproduction data. For the longitudinal part ($\alpha = 4$) we proceed analogously. For the interference terms we take the geometric mean of the transverse and longitudinal normalization factors. Accordingly the N_α are given by:

$$\begin{aligned}
\alpha = 0,\dots,3: &\quad N_\alpha = N_T = \tfrac{1}{2} \sum_{\substack{\lambda_\mathrm{V},\lambda_{\mathrm{N}'},\\ \lambda_\gamma=\pm1,\lambda_\mathrm{N}}} \left|T_{\lambda_\mathrm{V}\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_\mathrm{N}}\right|^2 ;\\
\alpha = 4 \quad\;\; : &\quad N_\alpha = N_\mathrm{L} = \sum_{\lambda_\mathrm{V},\lambda_{\mathrm{N}'},\lambda_\mathrm{N}} \left|T_{\lambda_\mathrm{V}\lambda_{\mathrm{N}'},0\lambda_\mathrm{N}}\right|^2 ;\\
\alpha = 5,\dots,8: &\quad N_\alpha = (N_\mathrm{T}N_\mathrm{L})^{\frac{1}{2}} .
\end{aligned} \tag{77}$$

Note that this implies $\mathrm{Tr}\,\rho^0 = \mathrm{Tr}\,\rho^4 = 1$. Appendix A lists the ρ^α expressed in terms of the helicity amplitudes.

Since we normalize the matrices ρ^α individually, the vector Π_α of eq. (74) differs of course from the corresponding Π_α which appears in the decomposition of $\rho(\gamma)$ (eq. (64)). We find:

$$\begin{aligned}
\Pi = \frac{1}{1+(\epsilon+\delta)R} \{ &1, -\epsilon\cos 2\Phi, -\epsilon\sin 2\Phi, \quad \frac{2m}{Q}(1-\epsilon)P_0 , \\
&(\epsilon+\delta)R , \sqrt{2\epsilon R(1+\epsilon+2\delta)}\cos\Phi , \quad \sqrt{2\epsilon R(1+\epsilon+2\delta)}\sin\Phi , \\
&\frac{2m}{Q}(1-\epsilon)\sqrt{R}\,(P_1\cos\Phi + P_2\sin\Phi) , \quad \frac{2m}{Q}(1-\epsilon)\sqrt{R}\,(P_1\sin\Phi - P_2\cos\Phi
\end{aligned} \tag{78}$$

where $R \equiv N_L/N_T = \sigma_L/\sigma_T$ and P_i are the components of the lepton polarization vector in the Breit system, see eq. (60).

As a consequence of parity conservation (eq. (72)) and of the symmetry properties of the Σ^α listed in table 1, the ρ^α obey the following symmetry relations:

$$\begin{aligned} \rho^\alpha_{-\lambda-\lambda'} &= (-1)^{\lambda-\lambda'} \rho^\alpha_{\lambda\lambda'}, \qquad && \alpha = 0, 1, 4, 5, 8\,; \\ \rho^\alpha_{-\lambda-\lambda'} &= -(-1)^{\lambda-\lambda'} \rho^\alpha_{\lambda\lambda'}, \qquad && \alpha = 2, 3, 6, 7 \end{aligned} \tag{79}$$

The proof of eq. (79) goes as follows:

$$\begin{aligned} \rho^\alpha_{-\lambda-\lambda'} &= \frac{1}{2N_\alpha} \sum_{\lambda_\gamma, \lambda'_\gamma} T_{-\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda-\lambda'_\gamma} \\ &= \frac{1}{2N_\alpha} \sum_{\lambda_\gamma\lambda'_\gamma} (-1)^{(\lambda-\lambda')+(\lambda_\gamma-\lambda'_\gamma)} T_{\lambda-\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'-\lambda'_\gamma} \\ &= \frac{1}{2N_\alpha} \sum_{\lambda_\gamma\lambda'_\gamma} (-1)^{(\lambda-\lambda')} T_{\lambda-\lambda_\gamma} C_\alpha \Sigma^\alpha_{-\lambda_\gamma-\lambda'_\gamma} T^*_{\lambda'-\lambda'_\gamma} \\ &= \frac{1}{2N_\alpha} C_\alpha (-1)^{\lambda-\lambda'} \sum_{\lambda_\gamma\lambda'_\gamma} T_{\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'\lambda'_\gamma} = C_\alpha (-1)^{\lambda-\lambda'} \rho^\alpha_{\lambda\lambda'}\,. \end{aligned} \tag{80}$$

Summation over the nucleon helicities is always understood. For the constant C_α, $C_\alpha = \pm 1$, see table 1. The symmetry relations eq. (79) reduce the number of independent matrix elements and leave us with two types of density matrices, which are listed in appendix B.

3.2. *Vector-meson decay angular distribution*

We express the vector-meson decay angular distribution in terms of the spin-one D functions and the vector-meson density matrix (see e.g. ref. [4]):

$$W(\cos\theta, \phi) = \frac{3}{4\pi} \sum_{\lambda,\lambda'} D^1_{\lambda 0}(\phi, \theta, -\phi)^* \rho(\mathrm{V})_{\lambda\lambda'} D^1_{\lambda' 0}(\phi, \theta, -\phi), \tag{81}$$

which can be written as

$$W(\cos\theta, \phi) = \sum_{\alpha=0}^{8} \Pi_\alpha W^\alpha(\cos\theta, \phi), \tag{82}$$

where W^α is obtained from eq. (81) replacing $\rho(V)$ by $\rho^\alpha(V)$. The symmetry relations, eqs. (79), greatly simplify the decay angular distribution. Making use of the trace condition $\mathrm{Tr}\,\rho^0 = \mathrm{Tr}\,\rho^4 = 1$ one has:

$$W^\alpha(\cos\theta,\phi) = \frac{3}{4\pi}\{\tfrac{1}{2}(1-\rho^\alpha_{00}) + \tfrac{1}{2}(3\rho^\alpha_{00}-1)\cos^2\theta$$

$$-\sqrt{2}\,\mathrm{Re}\,\rho^\alpha_{10}\sin 2\theta\cos\phi - \rho^\alpha_{1-1}\sin^2\theta\cos 2\phi\} \tag{83}$$

for $\alpha = 0, 4$;

$$W^\alpha(\cos\theta,\phi) = \frac{3}{4\pi}\{\rho^\alpha_{11}\sin^2\theta + \rho^\alpha_{00}\cos^2\theta$$

$$-\sqrt{2}\,\mathrm{Re}\,\rho^\alpha_{10}\sin 2\theta\cos\phi - \rho^\alpha_{1-1}\sin^2\theta\cos 2\phi\} \tag{84}$$

for $\alpha = 1, 5, 8$;

$$W^\alpha(\cos\theta,\Phi) = \frac{3}{4\pi}(\sqrt{2}\,\mathrm{Im}\,\rho^\alpha_{10}\sin 2\theta\sin\phi + \mathrm{Im}\,\rho^\alpha_{1-1}\sin^2\theta\sin 2\phi) \tag{85}$$

for $\alpha = 2, 3, 6, 7$.

3.2.1. Vector-meson decay angular distribution for unpolarized leptons

In the case of unpolarized leptons $P_0 = P_1 = P_2 = 0$ and consequently $\Pi_3 = \Pi_7 = \Pi_8 = 0$. This leads to

$$W^{\mathrm{unpol}}(\cos\theta,\phi,\Phi) = \frac{1}{1+(\epsilon+\delta)R}\,\frac{3}{4\pi}$$

$$\times\ [\tfrac{1}{2}(1-\rho^0_{00}) + \tfrac{1}{2}(3\rho^0_{00}-1)\cos^2\theta - \sqrt{2}\,\mathrm{Re}\,\rho^0_{10}\sin 2\theta\cos\phi - \rho^0_{1-1}\sin^2\theta\cos 2\phi$$

$$-\epsilon\cos 2\Phi\,\{\rho^1_{11}\sin^2\theta + \rho^1_{00}\cos^2\theta - \sqrt{2}\,\mathrm{Re}\,\rho^1_{10}\sin 2\theta\cos\phi - \rho^1_{1-1}\sin^2\theta\cos 2\phi\}$$

$$-\epsilon\sin 2\Phi\,\{\sqrt{2}\,\mathrm{Im}\,\rho^2_{10}\sin 2\theta\sin\phi + \mathrm{Im}\,\rho^2_{1-1}\sin^2\theta\sin 2\phi\}$$

$$+(\epsilon+\delta)R\,\{\tfrac{1}{2}(1-\rho^4_{00}) + \tfrac{1}{2}(3\rho^4_{00}-1)\cos^2\theta - \sqrt{2}\,\mathrm{Re}\,\rho^4_{10}\sin 2\theta\cos\phi$$

$$-\rho^4_{1-1}\sin^2\theta\cos 2\phi\} +$$

$$+\sqrt{2\epsilon R(1+\epsilon+2\delta)}\cos\Phi\,\{\rho^5_{11}\sin^2\theta+\rho^5_{00}\cos^2\theta$$

$$\sqrt{2}\,\mathrm{Re}\,\rho^5_{10}\sin 2\theta\cos\phi-\rho^5_{1-1}\sin^2\theta\cos 2\phi\}$$

$$+\sqrt{2\epsilon R(1+\epsilon+2\delta)}\sin\Phi\,\{\sqrt{2}\,\mathrm{Im}\,\rho^6_{10}\sin 2\theta\sin\phi+\mathrm{Im}\,\rho^6_{1-1}\sin^2\theta\sin 2\phi\}\,]\,. \tag{86}$$

Note that the contributions from $\rho^1, \rho^2, \rho^5, \rho^6$ can be separated in a measurement at a single value of ϵ since the functions $\cos\Phi$, $\cos 2\Phi$, $\sin\Phi$, $\sin 2\Phi$ are orthogonal to each other. In order to separate the contributions from ρ^0_{ik} and ρ^4_{ik} and to determine R, measurements at different ϵ values, i.e. at different values of the lepton scattering angle Θ for fixed Q^2, W are required.

The experiment measures in general 20 independent real quantities: σ_T and σ_L, the virtual photon cross sections, and the $18\,\rho^{\alpha}_{ik}$, $\alpha = 0-2, 4-6$ which appear in eq. (86) above (see also appendix B).

3.2.2. Vector-meson decay angular distribution for polarized leptons

3.2.2.1. Longitudinally polarized leptons ($\alpha_2 = 0, \pi$)

The components of Π which depend on the lepton polarization are (see also eqs. (60) and (78)):

$$\Pi_3=\frac{\sqrt{1-\epsilon^2}}{1+(\epsilon+\delta)R}P$$

$$\Pi_7=\frac{\sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))R}}{1+(\epsilon+\delta)R}P\cos\Phi\,,$$

$$\Pi_8=\frac{\sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))R}}{1+(\epsilon+\delta)R}P\sin\Phi\,, \tag{87}$$

where P is the degree of polarization. The decay angular distribution is given by:

$$W(\cos\theta,\phi,\Phi,\alpha_2=0,\pi)=W^{\text{unpol}}(\cos\theta,\phi,\Phi)\pm W^{\text{long pol}}(\cos\theta,\phi,\Phi)\,; \tag{88}$$

$$W^{\text{long pol}}(\cos\theta,\phi,\Phi)=\frac{1}{1+(\epsilon+\delta)R}\,\frac{3}{4\pi}$$

$$\times P\,[\sqrt{1-\epsilon^2}\,\{\sqrt{2}\,\mathrm{Im}\,\rho^3_{10}\sin 2\theta\sin\phi+\mathrm{Im}\,\rho^3_{1-1}\sin^2\theta\sin 2\phi\}\;+$$

$$+\sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))R}\cos\Phi\{\sqrt{2}\,\mathrm{Im}\,\rho^7_{10}\sin 2\theta\sin\phi$$

$$+\,\mathrm{Im}\,\rho^7_{1-1}\sin^2\theta\sin 2\phi\}$$

$$+\sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))R}\sin\Phi\ \{\rho^8_{11}\sin^2\theta+\rho^8_{00}\cos^2\theta$$

$$-\sqrt{2}\,\mathrm{Re}\,\rho^8_{10}\sin 2\theta\cos\phi-\rho^8_{1-1}\sin^2\theta\cos 2\phi\}]\,. \tag{89}$$

Provided the ratio R is known, a measurement of the decay angular distribution with a single setting of the lepton polarization and the lepton scattering angle is sufficient to determine all matrix elements ρ^{α}_{ik}, $\alpha = 0-8$ which enter eq. (88). Although the accuracy may be improved by taking data with both polarized und unpolarized beams, measurements with an unpolarized beam are not required in order to separate the matrix elements. This is because the angular functions multiplying the ρ^{α}_{ik} in eq. (88) – except for ρ^0_{ik} and ρ^4_{ik} – are orthogonal to each other. This remark might be important in view of the planned μ-beams at NAL and CERN II which in general will be polarized.

Besides the 20 independent real quantities obtained with unpolarized leptons, 8 additionals terms are accessible to measurements with longitudinal lepton polarization.

3.2.2.2. Transversely polarized leptons ($\alpha_2 = \frac{1}{2}\pi$)

In this case those terms of the decay angular distribution which depend on the lepton polarization carry a factor m/Q and are therefore important only when $Q^2 \approx m^2$. The corresponding Π components are,

$$\Pi_3 = 0\,;$$

$$\Pi_7 = \frac{(1-\epsilon)\sqrt{R}}{1+(\epsilon+\delta)R}\,\frac{2m}{Q}\,P\,\left(\sqrt{\frac{1-\epsilon}{1+\epsilon}}\cos\alpha_1\cos\Phi+\sin\alpha_1\sin\Phi\right);$$

$$\Pi_8 = \frac{(1-\epsilon)\sqrt{R}}{1+(\epsilon+\delta)R}\,\frac{2m}{Q}\,P\left(\sqrt{\frac{1-\epsilon}{1+\epsilon}}\cos\alpha_1\sin\Phi-\sin\alpha_1\cos\Phi\right).$$

Remember that α_1 is the angle between the plane of polarization and the lepton scattering plane. The matrix ρ^3 is not measurable since $\Pi_3 = 0$. The polarization dependent part of the decay distribution is given by:

$$W^{\text{trans pol}}(\cos\theta,\phi,\Phi) = \frac{1}{1+(\epsilon+\delta)R}\,\frac{3}{4\pi}\,(1-\epsilon)\sqrt{R}\,\frac{2m}{Q}\times$$

$$\times \Bigg[\left(\sqrt{\frac{1-\epsilon}{1+\epsilon}} \cos\alpha_1 \cos\Phi + \sin\alpha_1 \sin\Phi\right) \{\sqrt{2}\, \mathrm{Im}\, \rho^7_{10} \sin 2\theta \sin\phi$$

$$+ \mathrm{Im}\, \rho^7_{1-1} \sin^2\theta \sin 2\phi\}$$

$$+ \left(\sqrt{\frac{1-\epsilon}{1+\epsilon}} \cos\alpha_1 \sin\Phi - \sin\alpha_1 \cos\Phi\right) \{\rho^8_{11} \sin^2\theta + \rho^8_{00} \cos^2\theta$$

$$- \sqrt{2}\, \mathrm{Re}\, \rho^8_{10} \sin 2\theta \cos\phi - \rho^8_{1-1} \sin^2\theta \cos 2\phi\} \Bigg] . \tag{90}$$

As before the knowledge of R is required. At $\alpha_1 = 0$, due to the orthogonality of the angular functions the contributions from ρ^7, ρ^8 can be isolated for a given lepton polarization degree P. For arbitrary α_1 one must vary P.

It should be noted, however, that no additional information is obtained when transversely, in addition to longitudinally polarized leptons are used. Therefore and because of the factor m/Q in the transverse case it appears highly advantagous to use longitudinally polarized leptons. Fortunately the μ-beams obtainable at NAL and CERN II will be longitudinally polarized with a high degree of polarization. *.

3.2.3. Decay distribution when σ_T, σ_L are not separated

Separation of σ_T and σ_L requires measurements at different lepton scattering angles, which are difficult because of the inelastic scattering cross section decreasing rapidly with increasing Θ. If no separation is done the following matrix elements can be determined:

$$r^{04}_{ik} = \frac{\rho_{ik} + (\epsilon+\delta)R\, \rho^4_{ik}}{1+(\epsilon+\delta)R} , \qquad \alpha = 1-3 ;$$

$$r^{\alpha}_{ik} = \begin{cases} \dfrac{\rho^{\alpha}_{ik}}{1+(\epsilon+\delta)R} , & \\ & \\ \dfrac{\sqrt{R}\, \rho^{\alpha}_{ik}}{1+(\epsilon+\delta)R} , & \alpha = 5-8 . \end{cases} \tag{91}$$

Note: $\mathrm{Tr}\, r^{04} = 1$.

The decay distribution for polarized leptons reads now:

$$W^{\mathrm{unpol}}(\cos\theta, \phi, \Phi) = \frac{3}{4\pi} [\tfrac{1}{2}(1-r^{04}_{00}) + \tfrac{1}{2}(3r^{04}_{00} - 1)\cos^2\theta - \sqrt{2}\,\mathrm{Re}\, r^{04}_{10} \sin 2\theta \cos\phi$$

$$- r^{04}_{1-1} \sin^2\theta \cos 2\phi -$$

* We thank Dr. F.W. Brasse for a discussion on this point.

$$- \epsilon \cos 2\Phi \, \{r^1_{11} \sin^2\theta + r^1_{00} \cos^2\theta - \sqrt{2}\, \mathrm{Re}\, r^1_{10} \sin 2\theta \cos\phi - r^1_{1-1} \sin^2\theta \cos 2\phi\}$$

$$- \epsilon \sin 2\Phi \, \{\sqrt{2}\, \mathrm{Im}\, r^2_{10} \sin 2\theta \sin\phi + \mathrm{Im}\, r^2_{1-1} \sin^2\theta \sin 2\phi\}$$

$$+ \sqrt{2\epsilon(1+\epsilon+\delta)} \cos\Phi \, \{r^5_{11} \sin^2\theta + r^5_{00} \cos^2\theta - \sqrt{2}\, \mathrm{Re}\, r^5_{10} \sin 2\theta \cos\phi - r^5_{1-1} \sin^2\theta \cos 2\phi\}$$

$$+ \sqrt{2\epsilon(1+\epsilon+\delta)} \sin\Phi \, \{\sqrt{2}\, \mathrm{Im}\, r^6_{10} \sin 2\theta \sin\phi + \mathrm{Im}\, r^6_{1-1} \sin^2\theta \sin 2\phi\} \,] \,. \tag{92}$$

The additional part of the decay distribution arising from longitudinal lepton polarization is given by (see eqs. (88), (89)):

$$W^{\text{long pol}} = \frac{3}{4\pi} P \, [\sqrt{1-\epsilon^2} \, \{\sqrt{2}\, \mathrm{Im}\, r^3_{10} \sin 2\theta \sin\phi + \mathrm{Im}\, r^3_{1-1} \sin^2\theta \sin 2\phi\}$$

$$+ \sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))} \cos\Phi \, \{\sqrt{2}\, \mathrm{Im}\, r^7_{10} \sin 2\theta \sin\phi + \mathrm{Im}\, r^7_{1-1} \sin^2\theta \sin 2\phi\}$$

$$+ \sqrt{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))} \sin\Phi \, \{r^8_{11} \sin^2\theta + r^8_{00} \cos^2\theta - \sqrt{2}\, \mathrm{Re}\, r^8_{10} \sin 2\theta \cos\phi - r^8_{1-1} \sin^2\theta \cos 2\phi\} \,] \,. \tag{92a}$$

The matrix elements r^α_{ik} can be determined, for example, by moment analysis provided the full decay angular distribution is observed. For convenience the r^α_{ik} are listed in appendix C in terms of moments.

4. Natural and unnatural parity exchange

4.1. Separation of contributions from natural and unnatural parity exchange

Suppose we picture the reaction $\gamma_V N \to VN$ to proceed *via* t-channel exchange of particles (or particle systems) with pure natural ($P = (-1)^J$) or unnatural parity ($P = -(-1)^J$). Then measurements of the density matrices $\rho^0 - \rho^8$ allows one to separate the contributions from natural and unnatural parity exchange. This is possible because of a further symmetry property of the helicity amplitudes [15];

$$T(\theta^*_V)_{-\lambda_V\lambda_{N'},-\lambda_\gamma\lambda_N} = \pm(-1)^{\lambda_V-\lambda_\gamma} \, T(\theta^*_V)_{\lambda_V\lambda_{N'},\lambda_\gamma\lambda_N} , \tag{93}$$

which is valid to leading order in s. The + (–) sign applies to natural (unnatural) parity exchange. Writing T as a sum of contributions from natural and unnatural parity exchange

$$T = T^{\mathrm{N}} + T^{\mathrm{U}} , \tag{94}$$

we obtain:

$$T^{\binom{\mathrm{N}}{(\mathrm{U})}}(\theta_{\mathrm{V}}^{*})_{\lambda_{\mathrm{V}}\lambda_{\mathrm{N}'},\lambda_{\gamma}\lambda_{\mathrm{N}}} = \tfrac{1}{2}\left\{T(\theta_{\mathrm{V}}^{*})_{\lambda_{\mathrm{V}}\lambda_{\mathrm{N}'},\lambda_{\gamma}\lambda_{\mathrm{N}}} \pm (-1)^{\lambda_{\mathrm{V}}-\lambda}\, T(\theta_{\mathrm{V}}^{*})_{-\lambda_{\mathrm{V}}\lambda_{\mathrm{N}'},-\lambda_{\gamma}\lambda_{\mathrm{N}}}\right\} . \tag{95}$$

This relation in conjunction with the symmetry properties of Σ^{α} allows us to write

$$\rho_{\lambda\lambda'}^{\alpha} = \rho_{\lambda\lambda'}^{\alpha\mathrm{N}} + \rho_{\lambda\lambda'}^{\alpha\mathrm{U}} , \tag{96}$$

where the $\rho^{\alpha\mathrm{N}}$, $\rho^{\alpha\mathrm{U}}$ are defined as

$$\rho_{\lambda\lambda'}^{\alpha\binom{\mathrm{N}}{(\mathrm{U})}} = \frac{1}{2N^{\alpha}} \sum_{\substack{\lambda_{\gamma},\lambda'_{\gamma}\\ \lambda_{\mathrm{N}},\lambda_{\mathrm{N}'}}} T^{\binom{\mathrm{N}}{(\mathrm{U})}}_{\lambda\lambda_{\mathrm{N}'},\lambda_{\gamma}\lambda_{\mathrm{N}}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{\binom{\mathrm{N}}{(\mathrm{U})}*}_{\lambda'\lambda_{\mathrm{N}'},\lambda'_{\gamma}\lambda_{\mathrm{N}}} . \tag{97}$$

The essential content of eq. (96) is the absence of interference terms between T^{N} and T^{U} which holds for unpolarized nucleons. The proof goes as follows:

$$\begin{aligned}
&\sum T^{\mathrm{N}}_{\lambda\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*\mathrm{U}}_{\lambda'\lambda'_{\gamma}} \\
&= \sum_{\lambda_{\gamma}\lambda'_{\gamma}} \left\{T_{\lambda\lambda_{\gamma}} + (-1)^{\lambda-\lambda_{\gamma}}\, T_{-\lambda-\lambda_{\gamma}}\right\} \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}} \left\{T^{*}_{\lambda'\lambda'_{\gamma}} - (-1)^{\lambda'-\lambda'_{\gamma}}\, T^{*}_{-\lambda'-\lambda'_{\gamma}}\right\} \\
&= \sum_{\lambda_{\gamma}\lambda'_{\gamma}} \{T_{\lambda\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{\lambda'\lambda'_{\gamma}} - (-1)^{(\lambda-\lambda_{\gamma})+(\lambda'-\lambda'_{\gamma})}\, T_{-\lambda-\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{-\lambda'-\lambda'_{\gamma}} \\
&\qquad + (-1)^{\lambda-\lambda_{\gamma}}\, T_{-\lambda-\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{\lambda'\lambda'_{\gamma}} - (-1)^{\lambda'-\lambda'_{\gamma}}\, T_{\lambda\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{-\lambda'-\lambda'_{\gamma}}\} \\
&= \sum_{\lambda_{\gamma}\lambda'_{\gamma}} \{(-1)^{\lambda-\lambda_{\gamma}}\, T_{-\lambda-\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{\lambda'\lambda'_{\gamma}} - (-1)^{\lambda-\lambda_{\gamma}}\, T_{-\lambda-\lambda_{\gamma}}\, \Sigma^{\alpha}_{\lambda_{\gamma}\lambda'_{\gamma}}\, T^{*}_{\lambda'\lambda'_{\gamma}} = 0
\end{aligned} \tag{98}$$

Summation over the nucleon helicities is always understood. Note that the properties of Σ^α were not used to derive the result, which therefore holds for any matrix sandwiched between the T's.

Making use of the parity symmetry of the helicity amplitudes and of the second symmetry property of the matrices Σ^α (see table 1), namely

$$\Sigma^\alpha_{\lambda_\gamma - \lambda'_\gamma} = d_\alpha (-1)^{\lambda'_\gamma} \Sigma^\beta_{\lambda_\gamma \lambda'_\gamma} , \tag{99}$$

we shall express the $\rho^{\binom{\mathrm{N}}{\mathrm{U}}}$ as sums and differences of density matrix elements:

$$\begin{aligned}
\rho^{\alpha\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} &= \frac{1}{4N_\alpha} \sum_{\lambda_\gamma} (T_{\lambda\lambda_\gamma} \pm (-1)^{\lambda-\lambda_\gamma} T_{-\lambda-\lambda_\gamma}) \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} \\
&\quad \times (T^*_{\lambda'\lambda'_\gamma} \pm (-1)^{\lambda-\lambda'_\gamma} T^*_{-\lambda'-\lambda'_\gamma}) \\
&= \frac{1}{4N_\alpha} \sum_{\lambda_\gamma} \{T_{\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'\lambda'_\gamma} + (-1)^{(\lambda-\lambda_\gamma)+(\lambda'-\lambda'_\gamma)} \\
&\quad \times T_{-\lambda-\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{-\lambda'-\lambda'_\gamma} \\
&\quad \pm (-1)^{\lambda-\lambda_\gamma} T_{-\lambda-\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'\lambda'_\gamma} \pm (-1)^{\lambda'-\lambda'_\gamma} \\
&\quad \times T_{\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{-\lambda'-\lambda'_\gamma}\} \\
&= \frac{1}{2N_\alpha} \sum_{\lambda_\gamma} \{ T_{\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma\lambda'_\gamma} T^*_{\lambda'\lambda_\gamma} \pm (-1)^{\lambda'-\lambda'_\gamma} T_{\lambda\lambda_\gamma} \Sigma^\alpha_{\lambda_\gamma-\lambda'_\gamma} T^*_{-\lambda'\lambda'_\gamma}\} \\
&= \tfrac{1}{2}\{\rho^\alpha_{\lambda\lambda'} \pm (-1)^{\lambda'} T_{\lambda\lambda_\gamma} d_\alpha \Sigma^\beta_{\lambda_\gamma\lambda'_\gamma} T^*_{-\lambda'\lambda'_\gamma}\} \\
&= \tfrac{1}{2}\{\rho^\alpha_{\lambda\lambda'} \pm d_\alpha (-1)^{\lambda'} \rho^\beta_{\lambda-\lambda'}\} .
\end{aligned} \tag{100}$$

Summation over the nucleon helicities is always understood.

From this general result one obtains the relations listed in appendix D for the individual density matrices. Note that for the matrices ρ^4, ρ^5, ρ^8, a separation of the two parity contributions requires only the elements of the same matrix, namely ρ^4, ρ^5, ρ^8 respectively; note also that $\rho^{4U}_{00} = \rho^{5U}_{00} = \rho^{8U}_{00} = 0$.

Appendix D demonstrates the importance of measurements with longitudinally polarized leptons. It is only with these measurements that the N and U contributions to ρ^2, ρ^3 and $\rho^6 - \rho^8$ can be separated.

4.2. Natural and unnatural parity exchange cross sections

The cross section contributions from natural and unnatural parity exchange will be called σ^{N} and σ^{U}. According to appendix D

$$\sigma_{\mathrm{T}}^{\binom{\mathrm{N}}{\mathrm{U}}} = \tfrac{1}{2}\{1 \pm (2\rho^1_{1-1} - \rho^1_{00})\}\sigma_{\mathrm{T}} ,$$

$$\sigma_{\mathrm{L}}^{\binom{\mathrm{N}}{\mathrm{U}}} = \tfrac{1}{2}\{1 \mp (2\rho^4_{1-1} - \rho^4_{00})\}\sigma_{\mathrm{L}} . \tag{101}$$

Note that measurements with unpolarized leptons are sufficient to determine $\sigma^{\mathrm{N}}_{\mathrm{T}}$, $\sigma^{\mathrm{U}}_{\mathrm{T}}$ and $\sigma^{\mathrm{N}}_{\mathrm{L}}$, $\sigma^{\mathrm{U}}_{\mathrm{L}}$. The parity asymmetry, P_σ, defined as

$$P_\sigma = \frac{\sigma^{\mathrm{N}} - \sigma^{\mathrm{U}}}{\sigma^{\mathrm{N}} + \sigma^{\mathrm{U}}} , \tag{102}$$

for transverse and longitudinal photons, respectively, can be written as:

$$P\sigma_{\mathrm{T}} = 2\rho^1_{1-1} - \rho^1_{00} ,$$

$$P\sigma_{\mathrm{L}} = -(2\rho^4_{1-1} - \rho^4_{00}) . \tag{103}$$

5. The density matrices and the decay angular distribution in the case of *s*-channel helicity conservation and natural parity exchange

If the helicities are conserved in the hadronic c.m.s. (SCHC), i.e.

$$T_{\lambda_{\mathrm{V}}\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_{\mathrm{N}}} = T_{\lambda_{\mathrm{V}}\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_{\mathrm{N}}}\,\delta_{\lambda_{\mathrm{V}}\lambda_\gamma}\,\delta_{\lambda_{\mathrm{N}'}\lambda_{\mathrm{N}}} , \tag{104}$$

there are only three independent helicity amplitudes (see eq. (72)). The further assumption that only natural parity is being exchanged (eq. (93)) leaves us with two independent amplitudes, for which we choose $T_{1\frac{1}{2}1\frac{1}{2}}$ and $T_{0\frac{1}{2}0\frac{1}{2}}$.

Defining δ to be their relative phase,

$$T_{0\frac{1}{2}0\frac{1}{2}}\, T^*_{1\frac{1}{2}1\frac{1}{2}} \equiv |T_{0\frac{1}{2}0\frac{1}{2}}|\, |T_{1\frac{1}{2}1\frac{1}{2}}|\, e^{-i\delta}\ , \tag{105}$$

one finds for the density matrices the values listed in appendix E. In the limit $Q^2 \gg m^2$ the decay angular distribution for longitudinally polarized leptons is given by:

$$W(\cos\theta, \psi) = \frac{1}{1+\epsilon R}\, \frac{3}{8\pi}\, \{\sin^2\theta\, (1 + \epsilon \cos 2\psi) + 2\epsilon R \cos^2\theta$$

$$- \sqrt{2\epsilon(1+\epsilon)R}\, \cos\delta\, \sin 2\theta\, \cos\psi$$

$$+ \sqrt{2\epsilon(1-\epsilon)R}\, P \sin\delta\, \sin 2\theta\, \sin\psi\}\ , \tag{106}$$

where the polarization angle $\psi = \phi - \Phi$ has been introduced, and P is the degree of lepton polarization.

6. Summary

We have investigated vector-meson production by lepton scattering in the one-photon approximation. After deriving a standard decomposition of the photon spin density matrix for the general case of polarized incoming leptons the spin structure of vector-meson production by virtual photons has been studied. The vector-meson spin density matrix has been decomposed into a set of nine matrices ρ^α with convenient symmetry properties suitable for both an economic analysis of the experimentally determined decay distribution and a direct comparison with photoproduction results. The vector-meson decay distribution has been discussed for unpolarized and polarized leptons. As is well known, measurements at different lepton scattering angles, i.e. at different values of ϵ, are required to determine the ratio $R = \sigma_L/\sigma_T$. The lepton polarization does not help to get around this problem. However, the number of independent bilinear combinations of helicity amplitudes that can be measured from the decay distribution is increased from 20 to 28 when a longitudinally polarized instead of an unpolarized lepton beam is used. Transverse polarization appears to be of no practical interest. By use of longitudinal lepton polarization all observable density matrix elements can be split into contributions from natural and unnatural parity exchange in the t-channel. In this respect experiments with polarized lepton beams are definitely superior to those with unpolarized beams which permit this separation for only three of the nine matrices ρ^α.

We are indebted to Dr. S. Yellin for pointing out an error in the draft version.

Appendix A. *The matrices ρ^α expressed in terms of the helicity amplitudes*

For brevity, for the matrices $\rho^1, \rho^2 \ldots$ the summation over the nucleon helicities is not shown and the nucleon helicities are omitted. The normalization factors $N_\mathrm{T}, N_\mathrm{L}$ are defined as (see eq. (76)):

$$N_\mathrm{T} = \tfrac{1}{2} \sum_{\lambda\lambda_\gamma\lambda_{\mathrm{N}'}\lambda_\mathrm{N}} |T_{\lambda\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_\mathrm{N}}|^2 ;$$

$$N_\mathrm{L} = \sum_{\lambda,\lambda_{\mathrm{N}'}\lambda_\mathrm{N}} |T_{\lambda\lambda_{\mathrm{N}'},0\lambda_\mathrm{N}}|^2 .$$

$$\rho^0_{\lambda\lambda'} = \frac{1}{2N_\mathrm{T}} \sum_{\lambda_\gamma=\pm 1,\lambda_{\mathrm{N}'},\lambda_\mathrm{N}} T_{\lambda\lambda_{\mathrm{N}'}\lambda_\gamma\lambda_\mathrm{N}} T^*_{\lambda'\lambda_{\mathrm{N}'},\lambda_\gamma\lambda_\mathrm{N}} ,$$

$$\rho^1_{\lambda\lambda'} = \frac{1}{2N_\mathrm{T}} \sum_{\lambda_\gamma=\pm 1} T_{\lambda-\lambda_\gamma} T^*_{\lambda'\lambda_\gamma} ,$$

$$\rho^2_{\lambda\lambda'} = \frac{i}{2N_\mathrm{T}} \sum_{\lambda_\gamma=\pm 1} \lambda_\gamma T_{\lambda-\lambda_\gamma} T^*_{\lambda'\lambda_\gamma} ,$$

$$\rho^3_{\lambda\lambda'} = \frac{1}{2N_\mathrm{T}} \sum_{\lambda_\gamma=\pm 1} \lambda_\gamma T_{\lambda\lambda_\gamma} T^*_{\lambda'\lambda_\gamma} ,$$

$$\rho^4_{\lambda\lambda} = \frac{1}{N_\mathrm{L}} T_{\lambda 0} T^*_{\lambda' 0} ,$$

$$\rho^5_{\lambda\lambda'} = \frac{1}{\sqrt{2N_\mathrm{T}N_\mathrm{L}}} \sum_{\lambda_\gamma=\pm 1} \frac{\lambda_\gamma}{2} (T_{\lambda 0} T^*_{\lambda'\lambda_\gamma} + T_{\lambda\lambda_\gamma} T^*_{\lambda' 0}),$$

$$\rho^6_{\lambda\lambda'} = \frac{i}{\sqrt{2N_\mathrm{T}N_\mathrm{L}}} \sum_{\lambda_\gamma=\pm 1} \tfrac{1}{2} (T_{\lambda 0} T^*_{\lambda'\lambda_\gamma} - T_{\lambda\lambda_\gamma} T^*_{\lambda' 0}) ,$$

$$\rho^7_{\lambda\lambda'} = \frac{1}{\sqrt{2N_\mathrm{T}N_\mathrm{L}}} \sum_{\lambda_\gamma=\pm 1} \tfrac{1}{2} (T_{\lambda 0} T^*_{\lambda'\lambda_\gamma} + T_{\lambda\lambda_\gamma} T^*_{\lambda' 0}) ,$$

$$\rho^8_{\lambda\lambda'} = \frac{i}{\sqrt{2N_\mathrm{T}N_\mathrm{L}}} \sum_{\lambda_\gamma=\pm 1} \frac{\lambda_\gamma}{2} (T_{\lambda 0} T^*_{\lambda'\lambda_\gamma} - T_{\lambda\lambda_\gamma} T^*_{\lambda' 0}) .$$

Appendix B. *Form of the density matrices ρ^α*

The underlined elements are measurable from the decay distribution.

$$\rho^\alpha_{\lambda\lambda'} = \begin{pmatrix} \underline{\rho^\alpha_{11}} & \underline{\mathrm{Re}\,\rho^\alpha_{10}} + i\,\mathrm{Im}\,\rho^\alpha_{10} & \underline{\mathrm{Re}\,\rho^\alpha_{1-1}} \\ \underline{\mathrm{Re}\,\rho^\alpha_{10}} - i\,\mathrm{Im}\,\rho^\alpha_{10} & \underline{\rho^\alpha_{00}} & \underline{-\mathrm{Re}\,\rho^\alpha_{10}} - i\,\mathrm{Im}\,\rho^\alpha_{10} \\ \underline{\mathrm{Re}\,\rho^\alpha_{1-1}} & \underline{-\mathrm{Re}\,\rho^\alpha_{10}} + i\,\mathrm{Im}\,\rho^\alpha_{10} & \underline{\rho^\alpha_{11}} \end{pmatrix}$$

for $\alpha = 0, 1, 4, 5, 8$;

$$\rho^\alpha_{\lambda\lambda} = \begin{pmatrix} \rho^\alpha_{11} & \mathrm{Re}\,\rho^\alpha_{10} + \underline{i\,\mathrm{Im}\,\rho^\alpha_{10}} & \underline{i\,\mathrm{Im}\,\rho^\alpha_{1-1}} \\ \mathrm{Re}\,\rho^\alpha_{10} - i\,\mathrm{Im}\,\rho^\alpha_{10} & 0 & \mathrm{Re}\,\rho^\alpha_{10} + i\,\mathrm{Im}\,\rho^\alpha_{10} \\ \underline{-i\,\mathrm{Im}\,\rho^\alpha_{1-1}} & \mathrm{Re}\,\rho^\alpha_{10} - \underline{i\,\mathrm{Im}\,\rho^\alpha_{10}} & -\rho^\alpha_{11} \end{pmatrix}$$

for $\alpha = 2, 3, 6, 7$.

Appendix C. *Matrix elements r^α*

The matrix elements r^α_{ik} (see eq. 91)) for the case of longitudinally polarized leptons ($\alpha_2 = 0$) can be expressed in terms of moments with the definition

$$\langle F(\theta, \phi, \Phi) \rangle \equiv \frac{\int \mathrm{d}\cos\theta\, \mathrm{d}\phi\, \frac{\mathrm{d}\Phi}{2\pi}\, F(\theta, \phi, \Phi)\, W(\cos\theta, \phi, \Phi)}{\int \mathrm{d}\cos\theta\, \mathrm{d}\phi\, \frac{\mathrm{d}\Phi}{2\pi}\, W(\cos\theta, \phi, \Phi)} .$$

$$r^{04}_{00} = \tfrac{5}{2} \langle \cos^2\theta \rangle - \tfrac{1}{2} ,$$

$$\mathrm{Re}\, r^{04}_{10} = -\frac{5}{4\sqrt{2}} \langle \sin 2\theta \cos\phi \rangle ,$$

$$r^{04}_{1-1} = -\tfrac{5}{4} \langle \sin^2\theta \cos 2\phi \rangle ,$$

$$r^1_{00} = \frac{1}{\epsilon}\{\langle\cos 2\Phi\rangle - 5\langle\cos^2\theta\cos 2\Phi\rangle\}\ ,$$

$$r^1_{11} = \frac{1}{\epsilon}\{-\tfrac{3}{2}\langle\cos 2\Phi\rangle + \tfrac{5}{2}\langle\cos^2\theta\cos 2\Phi\rangle\}\ ,$$

$$\mathrm{Re}\, r^1_{10} = \frac{1}{\epsilon}\frac{5}{2\sqrt{2}}\langle\sin 2\theta\cos\phi\cos 2\Phi\rangle\, ,$$

$$r^1_{1-1} = \frac{1}{\epsilon}\tfrac{5}{2}\langle\sin^2\theta\cos 2\phi\cos 2\Phi\rangle\, ,$$

$$\mathrm{Im}\, r^2_{10} = -\frac{1}{\epsilon}\frac{5}{2\sqrt{2}}\langle\sin 2\theta\sin\phi\sin 2\Phi\rangle\, ,$$

$$\mathrm{Im}\, r^2_{1-1} = -\frac{1}{\epsilon}\tfrac{5}{2}\langle\sin^2\theta\sin 2\phi\sin 2\Phi\rangle\, ,$$

$$\mathrm{Im}\, r^3_{10} = \frac{1}{P\sqrt{1-\epsilon^2}}\frac{5}{4\sqrt{2}}\langle\sin 2\theta\sin\phi\rangle\, ,$$

$$\mathrm{Im}\, r^3_{1-1} = \frac{1}{P\sqrt{1-\epsilon^2}}\tfrac{5}{4}\langle\sin^2\theta\sin 2\phi\rangle\, ,$$

$$r^5_{00} = a\{-\langle\cos\Phi\rangle + 5\langle\cos^2\theta\cos\Phi\rangle\}\ ,$$

$$r^5_{11} = a\{\tfrac{3}{2}\langle\cos\Phi\rangle - \tfrac{5}{2}\langle\cos^2\theta\cos\Phi\rangle\}\ ,$$

$$\mathrm{Re}\, r^5_{10} = -a\frac{5}{2\sqrt{2}}\langle\sin 2\theta\cos\phi\cos\Phi\rangle\, ,$$

$$r^5_{1-1} = -a\tfrac{5}{2}\langle\sin^2\theta\cos 2\phi\cos\Phi\rangle\, ,$$

$$\mathrm{Im}\, r^6_{10} = a\frac{5}{2\sqrt{2}}\langle\sin 2\theta\sin\phi\sin\Phi\rangle\, ,$$

$$\mathrm{Im}\, r^6_{1-1} = a\tfrac{5}{2}\langle\sin^2\theta\sin 2\phi\sin\Phi\rangle\ ,$$

$$\mathrm{Im}\, r^7_{10} = \frac{1}{P}\, b\, \frac{5}{2\sqrt{2}}\langle\sin 2\theta\sin\phi\cos\Phi\rangle\, ,$$

$$\operatorname{Im} r^7_{1-1} = \frac{1}{P}\, b\, \tfrac{5}{2} \langle \sin^2\theta \sin 2\phi \cos\Phi \rangle ,$$

$$r^8_{00} = \frac{1}{P}\, b\, \{-\langle \sin\Phi \rangle + 5 \langle \cos^2\theta \sin\Phi \rangle\} ,$$

$$r^8_{11} = \frac{1}{P}\, b\, \{\tfrac{3}{2} \langle \sin\Phi \rangle - \tfrac{5}{2} \langle \cos^2\theta \sin\Phi \rangle\} ,$$

$$\operatorname{Re} r^8_{10} = -\frac{1}{P}\, b\, \frac{5}{2\sqrt{2}} \langle \sin 2\theta \cos\phi \sin\Phi \rangle ,$$

$$r^8_{1-1} = -\frac{1}{P}\, b\, \tfrac{5}{2} \langle \sin^2\theta \cos 2\phi \sin\Phi \rangle ;$$

$$a = \{2\epsilon(1+\epsilon+2\delta)\}^{-\frac{1}{2}} , \qquad b = \{2\epsilon(1-\epsilon)(1+2\delta/(1+\epsilon))\}^{-\frac{1}{2}} .$$

Appendix D. *Separation of contributions from natural (ρ^{N}) and unnatural parity ex change (ρ^{U}) in the t-channel*

$$\rho^{0\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}\,(\rho^0_{\lambda\lambda'} \mp (-1)^{\lambda'} \rho^1_{\lambda-\lambda'}) ,$$

$$\rho^{1\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^1_{\lambda\lambda'} \mp (-1)^{\lambda'} \rho^0_{\lambda-\lambda'}) , \qquad \rho^{5\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^5_{\lambda\lambda'} \pm (-1)^{\lambda'} \rho^5_{\lambda-\lambda'}) ,$$

$$\rho^{2\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^2_{\lambda\lambda'} \pm i(-1)^{\lambda'} \rho^3_{\lambda-\lambda'}) , \qquad \rho^{6\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^6_{\lambda\lambda'} \pm i(-1)^{\lambda'} \rho^7_{\lambda-\lambda'}) ,$$

$$\rho^{3\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^3_{\lambda\lambda'} \mp i(-1)^{\lambda'} \rho^2_{\lambda-\lambda'}) , \qquad \rho^{7\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^7_{\lambda\lambda'} \mp i(-1)^{\lambda'} \rho^6_{\lambda-\lambda'}) ,$$

$$\rho^{4\binom{\mathrm{N}}{\mathrm{U}}}_{\lambda\lambda'} = \tfrac{1}{2}(\rho^4_{\lambda\lambda'} \pm (-1)^{\lambda'} \rho^4_{\lambda-\lambda'}), \qquad \rho^{8\binom{\mathrm{N}}{\mathrm{U}}} = \tfrac{1}{2}(\rho^8_{\lambda\lambda'} \pm (-1)^{\lambda'} \rho^8_{\lambda-\lambda'}) .$$

Appendix E. *Vector-meson density matrices for the case of s-channel helicity conservation and natural parity exchange*

$$\rho^0 = \tfrac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\rho^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \rho^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\rho^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \rho^4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rho^5 = \frac{1}{\sqrt{8}}\begin{pmatrix} 0 & e^{i\delta} & 0 \\ e^{-i\delta} & 0 & -e^{-i\delta} \\ 0 & -e^{i\delta} & 0 \end{pmatrix} \qquad \rho^6 = \frac{1}{\sqrt{8}}\begin{pmatrix} 0 & -ie^{i\delta} & 0 \\ ie^{-i\delta} & 0 & ie^{-i\delta} \\ 0 & -ie^{i\delta} & 0 \end{pmatrix}$$

$$\rho^7 = \frac{1}{\sqrt{8}}\begin{pmatrix} 0 & e^{i\delta} & 0 \\ e^{-i\delta} & 0 & e^{-i\delta} \\ 0 & +e^{i\delta} & 0 \end{pmatrix} \qquad \rho^8 = \frac{1}{\sqrt{8}}\begin{pmatrix} 0 & -ie^{i\delta} & 0 \\ ie^{-i\delta} & 0 & -ie^{-i\delta} \\ 0 & ie^{i\delta} & 0 \end{pmatrix}$$

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